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On the behavior of F_2 and its logarithmic slopes

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Abstract

It is shown that the CKMT model for the nucleon structure function F_2, taken as the initial condition for the NLO evolution equations in perturbative QCD, provides a good description of the HERA data when presented in the form of the logarithmic slopes of F_2 vs x and Q^2 (Caldwell-plot), in the whole available kinematic ranges. Also the results obtained for the behavior of the gluon component of a nucleon are presented.

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1 The CKMT model

The CKMT model [1] for the parametrization of the nucleon structure function F_2 is a theoretical model based on Regge theory which provides a consistent formulation of this function in the region of low Q^2, and describes the experimental data on F_2 in that region.

The CKMT model [1] proposes for the nucleon structure functions

$$F_2(x,Q^2) = F_S(x,Q^2) + F_{NS}(x,Q^2), \tag{1}$$

the following parametrization of its two terms in the region of small and moderate Q^2. For the singlet term, corresponding to the Pomeron contribution:

$$F_S(x,Q^2) = A \cdot x^{-\Delta(Q^2)} \cdot (1-x)^{n(Q^2)+4} \cdot \left(\frac{Q^2}{Q^2+a}\right)^{1+\Delta(Q^2)}, \tag{2}$$

where the $x \to 0$ behavior is determined by an effective intercept of the Pomeron, Δ, which takes into account Pomeron cuts and, therefore (and this is one of the main points of the model), it depends on Q^2. This dependence was parametrized [1] as :

$$\Delta(Q^2) = \Delta_0 \cdot \left(1 + \frac{\Delta_1 \cdot Q^2}{Q^2+\Delta_2}\right). \tag{3}$$

Thus, for low values of Q^2 (large cuts), Δ is close to the effective value found from analysis of hadronic total cross-sections ($\Delta \sim 0.08$), while for high values of Q^2 (small cuts), Δ takes the bare Pomeron value, $\Delta \sim 0.2$-0.25. The parametrization for the non-singlet term, which corresponds to the secondary reggeon (f, A_2) contribution, is:

$$F_{NS}(x,Q^2) = B \cdot x^{1-\alpha_R} \cdot (1-x)^{n(Q^2)} \cdot \left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R}, \tag{4}$$

where the $x \to 0$ behavior is determined by the secondary reggeon intercept α_R, which is in the range α_R=0.4-0.5. The valence quark contribution can be separated into the contribution of the u (B_u) and d (B_d) valence quarks, the normalization condition for valence quarks fixes both contributions at one given value of Q^2 (we use $Q_v^2 = 2.GeV^2$ in our calculations). For both the singlet and the non-singlet terms, the behavior when $x \to 1$ is controlled by $n(Q^2)$, with $n(Q^2)$ being

$$n(Q^2) = \frac{3}{2} \cdot \left(1 + \frac{Q^2}{Q^2+c}\right), \tag{5}$$

so that, for Q^2=0, the valence quark distributions have the same power, given by Regge intercepts, as in the Quark Gluon String Model [2] or in the Dual Parton Model [3], $n(0)=\alpha_R(0)-\alpha_N(0) \sim 3/2$, and the behavior of $n(Q^2)$ for large Q^2 is taken to coincide with dimensional counting rules.

The total cross-section for real (Q^2=0) photons can be obtained from the structure function F_2 using the following relation:

$$\sigma_{\gamma p}^{tot}(\nu) = \left[\frac{4\pi^2 \alpha_{EM}}{Q^2} \cdot F_2(x,Q^2)\right]_{Q^2=0}. \tag{6}$$

The proper $F_2(x,Q^2) \sim Q^2$ behavior when $Q^2 \rightarrow 0$, is fulfilled in the model due to the last factors in equations 2 and 4. Thus, the $\sigma_{\gamma p}^{tot}(\nu)$ has the following form in the CKMT model:

$$\sigma_{\gamma p}^{tot}(\nu) = 4\pi^2 \alpha_{EM} \cdot \left(A \cdot a^{-1-\Delta_0} \cdot (2m\nu)^{\Delta_0} + (B_u + B_d) \cdot b^{-\alpha_R} \cdot (2m\nu)^{\alpha_R - 1} \right). \qquad (7)$$

The parameters were determined [1] from a joint fit of the $\sigma_{\gamma p}^{tot}$ data and the NMC data [4] on the proton structure function in the region $1GeV^2 \leq Q^2 \leq 5GeV^2$, and a very good description of the experimantal data available was obtained.

The next step in this approach is to introduce the QCD evolution in the partonic distributions of the CKMT model and thus to determine the structure functions at higher values of Q^2. For this, the evolution equation in two loops in the $\overline{\mathrm{MS}}$ scheme with $\Lambda = 200.MeV$ was used [1].

The results obtained by taking into account the QCD evolution in this way are [1] in a very good agreement with the experimental data on $F_2(x,Q^2)$ at high values of Q^2.

When the publication of the data [5, 6] on F_2 from HERA at low and moderate Q^2 provided the opportunity to include in the fit of the parameters of the model experimental points from the kinematical region where the CKMT parametrization should give a good description without need of any perturbative QCD evolution, one proceeded [7] to add these new data on F_2 from H1 and ZEUS at low and moderate Q^2, to those from NMC [4] and E665 [8] collaborations, and to data [9] on cross-sections for real photoproduction, into a global fit which allowed the test of the model in wider regions of x and Q^2. One took as initial condition for the values of the different parameters those obtained in the previous fit [1], and although the quality of the fit is not very sensitive to small changes in the values of the parameters, the best fit has been found for the values of the parameters given in Table1.

Table 1: Values of the parameters in the CKMT model obtained in the fit of F_2 when also the low Q^2 HERA data are included. All dimensional parameters are given in GeV^2. The valence counting rules provide the following values of B_u and B_d, for the proton case, when fixing their normalization at Q_v^2=2.GeV^2: B_u=1.1555, B_d=0.1722.

CKMT model	values of the parameters
A	0.1301
a	0.2628
Δ_0	0.09663
Δ_1	1.9533
Δ_2	1.1606
c	3.5489 (fixed)
b	0.3840
α_R	0.4150 (fixed)

The quality of the description provided by the CKMT model of all the experimental

data on $\sigma_{\gamma p}^{tot}$ and F_2, and, in particular, of the the new experimental data from HERA is very high, with a value of $\chi^2/d.o.f.$ for the global fit, $\chi^2/d.o.f.$=106.95/167, where the statistical and systematic errors have been treated in quadrature, and where the relative normalization among all the experimental data sets has been taken equal to 1.

Thus, by taking into account the general features of the CKMT model described above, we use the CKMT model to describe the experimental data in the region of low Q^2 ($0 < Q^2 < Q_0^2$), and then we take this parametrization as the initial condition at Q_0^2 to be used in the NLO QCD evolution equation to obtain F_2 at values of Q^2 higher than Q_0^2. In order to determine the distributions of gluons in a nucleon the CKMT model assumes [1] that the only difference between distributions of sea-quarks and gluons is in the $x \to 1$ behavior. Following [10] we write it in the form

$$xg(x,Q^2) = Gx\bar{q}(x,Q^2)/(1-x), \tag{8}$$

where $x\bar{q}(x,Q^2)$ is proportional to the expression in equation 2. The constant G is determined from the energy-momentum conservation sum rule.

We have performed our calculations for two different values of $Q_0^2 = 2.GeV^2$ and $Q_0^2 = 4.GeV^2$. We also show our results in the shape of both the $dF_2/dlnQ^2$ and the $dlnF2/dln(1/x)$ slopes in order to compare with the experimental data when given in the so-called Caldwell-plot. This approach provides a smooth transition from the region of small Q^2, which is governed by the physics of Regge theory, to a region of large Q^2, where the effects of QCD-evolution are important.

The way we proceed to calculate F_2, and its logarithmic derivatives $dF_2/dlnQ^2$, and $dlnF2/dln(1/x)$, is the following (see Appendix A for all the technical details on how the NLO QCD evolution has been performed):

- In the region $0 < Q^2 \leq Q_0^2$ we use the pure CKMT model for F_2.

- For $Q_0^2 < Q^2 \leq$ *charm threshold* [11], we make the QCD evolution of F_2 at NLO in the $\overline{\mathrm{MS}}$ scheme for a number of flavours $n_f = 3$, and we take as the starting parametrization that given by the CKMT model. For Q_0^2 we have used in this calculation two different values: $Q_0^2 = 2.GeV^2$, and $Q_0^2 = 4.GeV^2$.

- When *charm threshold* $< Q^2 \leq \bar{Q}^2 = 50.GeV^2$, also the QCD evolution of F_2 is implemented at NLO in the $\overline{\mathrm{MS}}$ scheme for a number of flavours $n_f = 3$, using the parton distribution functions for the u, d, s quarks, and by including the charm contribution via photon-gluon fusion.

- For values of $Q^2 > \bar{Q}^2$, QCD evolution is computed at NLO in the $\overline{\mathrm{MS}}$ scheme, but now with a number of flavours $n_f = 4$, and by using the parton distribution functions for the u, d, s, and c quarks.

One has to note that in the treatment of the charm contribution we have followed reference [12].

2 Results

The results we have obtained are presented in figures 1 to 9.

Figure 1 shows $F_2(x, Q^2)$ as a function of x for several values of Q^2, from $Q^2 = 0.6 GeV^2$ to $Q^2 = 17. GeV^2$. The dotted lines correspond to the pure CKMT model without any perturbative evolution, while the full lines run for the evoluted CKMT parametrization. When for a given value of Q^2 two full lines are depicted, the bold (solid) one has been obtained by taking the starting point for the QCD evolution as $Q_0^2 = 2. GeV^2$ ($Q_0^2 = 4. GeV^2$). Experimental points in this figure are from E665 [8], H1 [13], and ZEUS [14] collaborations.

In Figures 2.a and 2.b, we present the comparison of the pure CKMT parametrization of F_2 with the low Q^2 data of E665, ZEUS-BPC95, and ZEUS-BPT97, as compiled in [15] and [16]. One sees that the agreement between the CKMT model and the experimental data in this region of low Q^2 is good.

In Figure 3 (Caldwell-plot), the slope $dF_2/dlnQ^2$ is shown as a function of x, and compared with the $a + blnQ^2$ fit to the ZEUS F_2 data in bins of x. This plot was considered as an evidence for a transition from hard to soft regime of QCD in the region of $Q^2 \sim 5. GeV^2$ (see for example [17]). This question has been studied theoretically in references [18, 19]. Figure 3 shows that the CKMT model is in a good agreement with experimental points in the whole region of x and Q^2. One problem with the presentation of the data in Figure 3 is a strong correlation between x and Q^2 values for the data points. It follows from the formulas of CKMT model for $dF_2(x, Q^2)/dlnQ^2$ given in Appendix B that for a fixed value of Q^2 this quantity monotonically increases as $x \to 0$. The existence of a maximun of $dF_2(x, Q^2)/dlnQ^2$ in Figure 3 is related to the correlation between Q^2 and x in the region of small x (or Q^2). The same conclusion was achieved in reference [18], and recently confirmed by experimental data [16].

Figures 4 and 5 show the slope $dlnF2/dln(1/x)$ as a function of Q^2 compared to the fits $F_2 = Ax^{-\Delta_{eff}}$ of the the ZEUS [14] and H1 [13] data, respectively. In Figure 4, as the x range of the BPC95 data is restricted, also the E665 [8] data were included in [14], and are now also taken into account. This slope is sometimes interpreted as the Δ_{eff} of the Pomeron exchange, $\Delta_{eff} = dlnF2/dln(1/x)$. Let us note that in our approach Δ_{eff} for $Q^2 > Q_0^2$ can not be interpreted as an effective Pomeron intercept, because the QCD evolution leads to a substantial increase of Δ_{eff} as Q^2 increases. On the other hand this effect should decrease as $x \to 0$.

In the experimental fits, each Q^2 bin corresponds to a average value of x, $< x >$, calculated from the mean value of $ln(1/x)$ weighted by the statistical errors of the corresponding F_2 values in that bin. Even though we can proceed as in the experimental fits, and we get a very good agreement with the data, since the estimation of $< x >$ is in some sense artificial and arbitrary, and it introduces unphysical wiggles when drawing one full line connecting the different bins, we made for all the Q^2 bins in this figures the choice of the smallest x in the data, instead of considering a different $< x >$ for each Q^2. This choice is based on the fact that the ansatz $\Delta_{eff} = dlnF2/dln(1/x)$ is actually valid for small x, and it results in a smooth curve except for the jump in the region around $Q^2 \sim 50. GeV^2$, where the evolution procedure changes (again, see Appendix A for more details).

Since the structure function F_2 in the region of low x is determined at large extent by the gluon component, we present our prediction for the behavior of this gluon component. Thus, Figure 6 shows the gluon density distribution as a function of Q^2 calculated by performing the NLO QCD evolution of the CKMT model, and its comparison with the

H1 Collaboration data in reference [20], and Figure 7 represents the gluon densities at $\mu^2 = 25. GeV^2$ as a function of x calculated by evoluting the CKMT model at NLO in the QCD evolution, and compared to those determined from H1 DIS and photoproduction data. Experimental data on D^* meson cross-section measurements are from references [16, 20]. Figure 8 shows the behavior of $xg(x, \mu^2)$ at $\mu^2 = 200\ GeV^2$ as a function of Q^2 to be compared with the H1-dijets results [16, 21]. Finally, Figure 9 shows the prediction of the CKMT model for $xg(x, Q^2)$ as a function of x at the values of Q^2 measured both by H1 and ZEUS collaborations.

A satisfactory agreement with the experiment is obtained in the whole ranges of x and Q^2 where experimental data are available, showing that the experimental behavior of F_2, its logarithmic slopes, and its gluon component can be described by using as initial condition for the QCD evolution equation a model of F_2 where the shadowing effects which are important at low values of Q^2 are included, like the CKMT model.

3 Conclusions

The CKMT model for the parametrization of the nucleon structure functions provides a very good description of all the available experimental data on $F_2(x, Q^2)$ at low and moderate Q^2, including the recent small-x HERA points.

An important ingredient of the model is the dependence of an effective intercept of the Pomeron on Q^2. It has been shown recently [22] that such a behavior is naturally reproduced in a broad class of models based on reggeon calculus, which describes simultaneously the structure function F_2 and the diffractive production by virtual photons.

Use of the CKMT model as the initial condition for the QCD-evolution equations in the region of $Q^2 = 2. \div 5. GeV^2$ leads to a good description of all available data in a broad region of Q^2, including the logarithmic slopes of the structure function $F_2(x, Q^2)$, $dF_2(x, Q^2)/dlnQ^2$ and $dlnF_2(x, Q^2)/dln(1/x)$. Thus an unified description of the data on F_2 for all values of Q^2 is achieved.

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Appendix A – NLO QCD evolution of $F_2(x, Q^2)$

For the reader convenience we present here some technical remarks concerning the NLO QCD calculation of $F_2(x, Q^2)$.

For sufficiently large $Q^2 > 1\ GeV^2$, the structure function $F_2(x, Q^2)$ can be expressed by perturbative parton distributions. In leading order (LO) perturbation theory, the expression is given as

$$\frac{1}{x}F_2(x, Q^2) = x \sum_q e_q^2 \{q(x, Q^2) + \bar{q}(x, Q^2)\}, \tag{9}$$

where q and $\bar{q}$ denote the quark and anti-quark distribution functions, e_q^2 the square of the quark electric charge, and the sum runs over all quark flavors included [11]. On the other hand, with $F_2(x, Q^2)$ given in eqs. (1-5), and making reasonable assumptions concerning the flavor structure of the QCD-sea, one can extract from $F_2(x, Q^2)$ the different parton distribution functions, including that of the gluon component [1]. Generally, the calculation of $F_2(x, Q^2)$ at $Q^2 \gg 1\ GeV^2$ requires a Q^2-evolution à la DGLAP [23]. The procedure consists in the solution of the LO-DGLAP equations for the parton distribution functions using reasonable initial distributions at a starting value $Q^2 = Q_0^2$ ($1\ GeV^2 < Q_0^2 < 5\ GeV^2$). Using eq.(9), the resulting quark distributions at Q^2 can be recombined to F_2 at this virtuality.

By the evolution of the CKMT-model we mean the application of this procedure to the model discussed in this paper. As mentioned above, the CKMT-model of $F_2(x, Q^2)$ is valid within $0 \leq Q^2 < 5\ GeV^2$. Due to the good aggreement with experimental data the parton distributions extracted from F_2^{CKMT} at a Q_0^2 in the range given above seem to be reasonable initial distributions for an evolution to higher Q^2.

In next to leading order (NLO), the relation between $F_2(x, Q^2)$ and the parton distribution functions is more complicated and depends on the renormalization scheme. The calculations presented here are performed in the $\overline{\text{MS}}$-scheme [24]. In this context, the structure function is given by [11] as

$$\frac{1}{x}F_2(x, Q^2) = \sum_q e_q^2 \left\{ q(x, Q^2) + \bar{q}(x, Q^2) + \frac{\alpha_s(Q^2)}{2\pi}\left[C_{q,2} * (q + \bar{q}) + 2 \cdot C_{g,2} * g\right] \right\}, \tag{10}$$

where $q, \bar{q}$ and g are the NLO quark, anti-quark and gluon distribution functions, respectively. α_s denotes the strong coupling constant in NLO. The convolutions $C * q$ and $C * g$ are defined as

$$C * q = \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}\right) q(y, Q^2). \tag{11}$$

The Wilson coefficients $C_{q,g,2}(z)$ are given by

$$\begin{aligned} C_{q,2}(z) &= \frac{4}{3}\left[\frac{1+z^2}{1-z}\left(\ln\frac{1-z}{z} - \frac{3}{4}\right) + \frac{1}{4}(9+5z)\right]_+ , \\ C_{g,2}(z) &= \frac{1}{2}\left[(z^2 + (1-z)^2)\ln\frac{1-z}{z} - 1 + 8z(1-z)\right]. \end{aligned} \tag{12}$$

Here, the integral over a $[\cdot]_+$-distribution is defined as described in [25]:

$$\begin{aligned} C_+ * q &= \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}\right)_+ q(y,Q^2) \\ &= \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}\right)\left[q(y,Q^2) - \frac{x}{y} q(x,Q^2)\right] - q(x,Q^2)\int_0^x dy C(y). \end{aligned} \tag{13}$$

There are alternative renormalization schemes as, for instance, the DIS-scheme [11]. Here, the form of eq. (9) is kept for NLO also, i.e.

$$\frac{1}{x}F_2(x,Q^2) = x\sum_q e_q^2\{q_{DIS}(x,Q^2) + \bar{q}_{DIS}(x,Q^2)\}. \tag{14}$$

The relation between the $\overline{MS}$- and the DIS-distributions is given by

$$\begin{aligned} \overset{(-)}{q}_{DIS}(x,Q^2) &= \overset{(-)}{q}(x,Q^2) + \frac{\alpha_s(Q^2)}{2\pi}\left[C_{q,2} * \overset{(-)}{q} + C_{g,2} * g\right] + O(\alpha_s^2), \\ g_{DIS}(x,Q^2) &= g(x,Q^2) - \frac{\alpha_s(Q^2)}{2\pi}\left[\sum_q C_{q,2} * (q+\bar{q}) + 2f \cdot C_{g,2} * g\right] + O(\alpha_s^2). \end{aligned} \tag{15}$$

The parameter f denotes the number of active flavors in the sea.

Our procedure to extract the parton-distributions from F_2^{CKMT} is based on the LO-formula eq. (9). Therefore, in NLO, we extract the DIS-distributions. Now, the task is to calculate the $\overline{MS}$-distributions at $Q^2 = Q_0^2$. This can be done using a first order approximation in $\alpha_s(Q^2)/2\pi$:

$$\begin{aligned} \overset{(-)}{q}(x,Q_0^2) &\approx \overset{(-)}{q}_{DIS}(x,Q_0^2) - \frac{\alpha_s(Q_0^2)}{2\pi}\left[C_{q,2} * \overset{(-)}{q}_{DIS} + C_{g,2} * g_{DIS}\right] \\ g(x,Q_0^2) &\approx g_{DIS}(x,Q_0^2) + \frac{\alpha_s(Q_0^2)}{2\pi}\left[\sum_q C_{q,2} * (q_{DIS} + \bar{q}_{DIS}) + 2f \cdot C_{g,2} * g_{DIS}\right]. \end{aligned} \tag{16}$$

In summary, the Q^2-evolution of F_2^{CKMT} works as follows:

1. One chooses an appropriate value $Q^2 = Q_0^2 > 1\ GeV^2$ as a starting point for the evolution. These are $Q_0^2 = 2\ GeV^2$ or $Q_0^2 = 4\ GeV^2$ in our calculations.

2. At $Q^2 = Q_0^2$, one extracts the NLO parton distribution functions from F_2^{CKMT} . The relation between these parton distributions and the structure function is given by eq.(14) which is formally the same as eq.(9) in LO. So the resulting parton distributions are the DIS-functions, i.e. $q_{DIS}(x,Q_0^2)$, $\bar{q}_{DIS}(x,Q_0^2)$ and $g_{DIS}(x,Q_0^2)$.

3. Using eq.(16) one calculates the $\overline{MS}$-distributions $q(x,Q_0^2)$, $\bar{q}(x,Q_0^2)$ and $g(x,Q_0^2)$.

4. These $\overline{MS}$-functions serve as initial distributions in a numerical procedure to solve the NLO-DGLAP-equations in the $\overline{MS}$-scheme for a certain value $Q^2 > Q_0^2$. The result are the evoluted $\overline{MS}$-parton distributions $q(x,Q^2)$, $\bar{q}(x,Q^2)$ and $g(x,Q^2)$.

5. Finally, using eq.(10), the structure function $F_2^{CKMT}(x,Q^2)$ can be recalculated.

The charm production is of particular interest. Following refs. [11, 12], the assumption of a "massless" charm quark produced above the threshold $Q_c^2 = 4m_c^2$ (m_c^2 – charm quark mass) via the usual DGLAP-evolution is not realistic. This procedure is useful in the range of high $Q^2 \gg Q_c^2$ only. In the intermediate region $Q_c^2 < Q^2 < \bar{Q}^2 = 50\ GeV^2$, the charm is treated via a photon-gluon fusion process. The corresponding contribution to the structure function is defined as

$$\frac{1}{x} F_2^c(x, Q^2, m_c^2) \;=\; 2e_c^2 \,\frac{\alpha_s(\mu^2)}{2\pi} \int_{ax}^{1} \frac{dy}{y} \;\cdot C_{g,2}^c\left(\frac{x}{y}, \frac{m_c^2}{Q^2}\right) \cdot g(y, \mu^2), \tag{17}$$

where $\mu^2 = 4m_c^2$, $a = 1 \;+\; 4m_c^2/Q^2$, and, the coefficient $C_{g,2}^c(Z, R)$ is given by

$$\begin{aligned} C_{g,2}^c(Z, R) \;&=\; \frac{1}{2}\Big\{[Z^2 + (1-Z)^2 + 4ZR(1-3Z) - 8Z^2R^2] \ln \frac{1+V}{1-V} \\ &+\; V[-1 + 8Z(1-Z) - 4ZR(1-Z)]\Big\}, \end{aligned} \tag{18}$$

with $V^2 = 1 - 4RZ/(1-Z)$. So, F_2 in total is given by eq.(10) where the sum runs over $q = u, d, s$ plus eq.(17). The contributions of the botom and top quarks are neglected here. Precisely, the charm threshold is defined as discussed in refs. [11, 12],

$$W^2 \equiv Q^2(1/x \;-\; 1) \geq Q_c^2 = 4m_c^2. \tag{19}$$

The Q^2-dependence of F_2 can be summarized as follows:

i) $Q^2 < Q_0^2$**:** In the low Q^2 region, F_2 is calculated as given in the pure CKMT-model, eqs.(1–5).

ii) $Q_0^2 < Q^2 < Q_c^2$**:** Below the charm threshold F_2 is calculated using eq.(10) from NLO QCD evoluted parton distributions in the $\overline{MS}$-scheme. The number of flavors is $f = 3$ (u, d, s).

iii) $Q_c^2 < Q^2 < \bar{Q} = 50\ GeV^2$**:** Above the charm threshold F_2 is determined from eqs.(10) and (17). Note that the number of flavors active in the evolution is again $f = 3$ (u, d, s). However, $f = 4$ after the charm is produced. This is important for the calculation of α_s.

iv) $Q^2 > \bar{Q}^2 = 50\ GeV^2$**:** In the high Q^2-region, F_2 is given by eq.(10). The charm is produced as "massless" quark in the evolution process. Generally, the number of flavors is $f = 4$.

The threshold $\bar{Q}^2$ where the charm production in the evolution process is more important than the photon-gluon fusion is discussed in detail in [12]. The value of 50 GeV^2 is chosen to guarantee a transition as smooth as possible. This method works better for $x \to 0$ than for $x \to 1$. This explains the small wiggles in some of the figures at $Q^2 = 50\ GeV^2$.

Appendix B – The slopes of $F_2(x, Q^2)$

For low Q^2, the "pure" CKMT-model is used, i.e. the one defined in eqs.(1-5). Here, the calculation of the slopes as $dF_2(x,Q^2)/d\ln Q^2$ and $d\ln F_2(x,Q^2)/d\ln(1/x) = \Delta_{eff}$ is straightforward. Considering x and Q^2 as independent variables one gets

$$\begin{aligned}\frac{dF_2(x,Q^2)}{dlnQ^2} = \quad & F_S(x,Q^2) \quad [\tfrac{\Delta_2}{Q^2+\Delta_2}\,(\Delta(Q^2)-\Delta_0)\,ln\tfrac{Q^2}{x(Q^2+a)} \\ & \qquad +\tfrac{c}{Q^2+c}\left(n(Q^2)-\tfrac{3}{2}\right)ln(1-x)+\tfrac{a\,\left(1+\Delta(Q^2)\right)}{Q^2+a}] \\ + \quad & F_{NS}(x,Q^2) \quad [\tfrac{c}{Q^2+c}\left(n(Q^2)-\tfrac{3}{2}\right)ln(1-x)+\tfrac{b\ \alpha_R(0)}{Q^2+b}],\end{aligned} \tag{20}$$

which in the limit $Q^2 \to 0$ takes the form

$$\frac{dF_2(x,Q^2)}{dlnQ^2} \sim (1+\Delta_0)\,F_S(x,Q^2)+\alpha_R(0)F_{NS}(x,Q^2). \tag{21}$$

Also, if one considers the case when W is fixed one can take $x \sim C \cdot Q^2$, and then, up to constant factors, one gets:

$$\begin{aligned}\frac{dF_2(x,Q^2)}{dlnQ^2} = \quad & F_S(x,Q^2) \quad [-\tfrac{\Delta_2}{Q^2+\Delta_2}\,(\Delta(Q^2)-\Delta_0)\,ln(Q^2+a) \\ & \qquad -\Delta(Q^2)+\tfrac{c}{Q^2+c}\left(n(Q^2)-\tfrac{3}{2}\right)ln(1-Q^2) \\ & \qquad -\tfrac{Q^2 n(Q^2)}{1-Q^2}+\tfrac{a\ (1+\Delta(Q^2))}{Q^2+a}] \\ + \quad & F_{NS}(x,Q^2) \quad [\tfrac{c}{Q^2+c}(n(Q^2)-\tfrac{3}{2})ln(1-Q^2) \\ & \qquad +\tfrac{b\ \alpha_R(0)}{Q^2+b}+(1-\alpha_R(0))-\tfrac{Q^2 n(Q^2)}{1-Q^2}].\end{aligned} \tag{22}$$

Now, if one takes W fixed with $Q^2 \sim x \to 0$, one can easily see that this equation simply reduces to:

$$\frac{dF_2(x,Q^2)}{dlnQ^2} \sim F_2(x,Q^2). \tag{23}$$

The calculations presented in the paper are based on the assumption of independent x and Q^2, i.e. eqs.(20, 21). In this context, the effective x-slope $\Delta_{eff} = d\ln F_2(x,Q^2)/d\ln(1/x)$ is given by

$$\begin{aligned}F_2(x,Q^2)\cdot\frac{d\ln F_2(x,Q^2)}{d\ln(1/x)} = \quad & [\Delta(Q^2)+\tfrac{x}{1-x}(n(Q^2)+4)]\cdot F_S \\ + \quad & [\alpha_R(0)-1+\tfrac{x}{1-x}n(Q^2)+\tfrac{x\ B_d}{B_u+B_d(1-x)}]\cdot F_{NS}.\end{aligned} \tag{24}$$

For $Q^2 > Q_0^2$, the slopes have to be calculated from the evoluted structure function. Here, there are two fundamental procedures, the pure numerical and the mainly analytical calculations. The pure numerical procedure is very simple:

$$\frac{dF_2(x,Q^2)}{d\ln Q^2} \approx Q^2\cdot\tfrac{1}{2\delta Q^2}\cdot[F_2(x,Q^2+\delta Q^2)-F_2(x,Q^2-\delta Q^2)], \tag{25}$$

$$\frac{d\ln F_2(x,Q^2)}{d\ln(1/x)} \approx (-1)\cdot\frac{x}{F_2(x,Q^2)}\cdot\tfrac{1}{2\delta x}\cdot[F_2(x+\delta x,Q^2)-F_2(x-\delta x,Q^2)]. \tag{26}$$

$F_2(x,Q^2)$ is the evoluted structure function whereas δQ^2 and δx denote the corresponding increments which are fixed to be $10^{-3}\cdot Q^2$ or $10^{-3}\cdot x$ in the calculations presented. For low Q^2, we have checked this procedure comparing the values of eqs. (25,26) with those calculated using eqs. (20,24). The agreement is very good which, in some cases, is demonstrated by identical numbers. This numerical procedure is the method used to determine the effective x-slope $\Delta_{eff} = d\ln F_2(x,Q^2)/d\ln(1/x)$ of the evoluted structure function. In the case of $dF_2(x,Q^2)/d\ln Q^2$ there is, in addition, the way of mainly analytical calculations. If the parton distribution functions are known their derivatives concerning Q^2 can be calculated from the DGLAP-equations. Instead of Q^2 the parameter

$$S = \ln\left\{\tfrac{T}{T_o}\right\}, \quad T = \ln(Q^2/\Lambda^2_{QCD}), \quad T_o = \ln(Q_0^2/\Lambda^2_{QCD}) \tag{27}$$

is often used in perturbation theory. In terms of S

$$\frac{dF_2}{d\ln Q^2} = \frac{1}{\ln(Q^2/\Lambda^2_{QCD})}\frac{dF_2}{dS}, \tag{28}$$

and in the $\overline{MS}$-scheme

$$\begin{aligned}\frac{1}{x}\frac{dF_2(x,S)}{dS} &= \sum_q e_q^2\Bigg\{\frac{dq(x,S)}{dS}+\frac{d\bar{q}(x,S)}{dS} \\ &\quad +\frac{\alpha_s(Q^2)}{2\pi}\left[C_{q,2}*(\frac{dq}{dS}+\frac{d\bar{q}}{dS})+2\cdot C_{g,2}*\frac{dg}{dS}\right] \\ &\quad +\frac{1}{2\pi}\frac{d\alpha_s(Q^2)}{dS}\left[C_{q,2}*(q+\bar{q})+2\cdot C_{g,2}*g\right]\Bigg\}.\end{aligned} \tag{29}$$

The numerical integration procedure for solving the DGLAP-equations used in the work presented here gives the evoluted parton distributions and their derivatives on S as the output. In NLO, $d\alpha_s(Q^2)/dS$ is simple to calculate,

$$\begin{aligned}\frac{\alpha_s(T)}{2\pi} &= \frac{2}{\beta_0 T}\left(1-\frac{\beta_1}{\beta_0^2}\frac{\ln(T)}{T}\right), \\ \frac{1}{2\pi}\frac{d\alpha_s(T)}{dT} &= -\frac{1}{T}\cdot\frac{\alpha_s(T)}{2\pi}+\frac{2\beta_1}{\beta_0^2 T^3}(\ln(T)-1), \\ \frac{1}{2\pi}\frac{d\alpha_s}{dS} &= T\cdot\frac{1}{2\pi}\frac{d\alpha_s}{dT}.\end{aligned} \tag{30}$$

Thus, with the derivatives dq/dS, $d\bar{q}/dS$ and dg/dS one gets the Q^2-derivative of F_2. This method is called as "mainly analytical" (it includes a numerical integration procedure).

Eq.(29) is valid below the charm threshold [11, 12], i.e. $Q_0^2 < Q^2 < Q_c^2$, and in the high Q^2-region where the charm can be considered as a "massless" dynamical quark [12]. As described above, the charm is treated via a photon-gluon fusion process in the range $Q_c^2 < Q^2 < \bar{Q}^2 = 50\ GeV^2$ [11, 12]. From eq. (17) the charm slope contribution can be detrmined as

$$\frac{1}{x}\frac{dF_2^c(x,Q^2,m_c^2)}{d\ln Q^2} = 2e_c^2\,\frac{\alpha_s(\mu^2)}{2\pi}\int_{ax}^{1}\frac{dy}{y}\cdot\frac{dC^c_{g,2}}{dlnQ^2}\left(\frac{x}{y},\frac{m_c^2}{Q^2}\right)\cdot g(y,\mu^2). \tag{31}$$

The total slope is the sum of eqs. (29) and (31).

We have calculated the Q^2-slope of the evoluted F_2 in the perturbative region ($Q^2 \geq Q_0^2$) using both, the numerical and the analytical methods. The values are in agreement although the difference increases somewhat in the region near to Q_0^2. The values presented in the figures are from the numerical calculation.

Figure captions

Figure 1. F_2 as a function of x computed in the CKMT model for twelve different values of Q^2, and compared with the following experimental data (see [14] for the experimental references): ZEUS SVX95 (black circles), H1 SVX95 (white triangles), ZEUS BPC95 (white squares), E665 (white diamonds), and ZEUS 94 (white circles). The dotted line is the theoretical result obtained with the pure CKMT model, and the bold (solid) line is the result obtained with the NLO QCD-evoluted CKMT model when one takes $Q_0^2 = 2.GeV^2$ ($Q_0^2 = 4.GeV^2$).

Figure 2. F_2 as a function of x computed in the CKMT model for six (a) and five (b) different low values of Q^2, and compared with the following experimental data (see [14, 15, 16] for the experimental references): ZEUS BPT97 (black circles), ZEUS BPC95 (white circles), and E665 (white squares). The theoretical result has been obtained with the pure CKMT model.

Figure 3. $dF_2/dlnQ^2$ as a function of x computed by performing the NLO QCD perturbative evolution of the CKMT model (see appendices A and B for details on the calculation), and compared with the fit of the ZEUS F_2 data in bins of x to the form $a + blnQ^2$ (see reference [14] and references therein for more details on the data and the experimental fit). The dotted line is the theoretical result obtained with the pure CKMT model, and the bold (solid) line is the result obtained with the NLO QCD-evoluted CKMT model when one takes $Q_0^2 = 2.GeV^2$ ($Q_0^2 = 4.GeV^2$).

Figure 4. $dlnF_2/dln(1/x)$ as a function of Q^2 calculated by performing the NLO QCD evolution of the CKMT model, and compared to the fit $F_2 = Ax^{-\Delta_{eff}}$ of the ZEUS [14] and the E665 [8] data with $x < 0.01$. For details on the CKMT calculation, see Appendices A and B. The dotted line is the theoretical result obtained with the pure CKMT model, and the bold (solid) line is the result obtained with the NLO QCD-evoluted CKMT model when one takes $Q_0^2 = 2.GeV^2$ ($Q_0^2 = 4.GeV^2$).

Figure 5. $dlnF_2/dln(1/x)$ as a function of Q^2 calculated by performing the NLO QCD evolution of the CKMT model, and compared to the fit $F_2 = Ax^{-\Delta_{eff}}$ of the H1 data [13]. For details on the CKMT calculation, see Appendices A and B. The dotted line is the theoretical result obtained with the pure CKMT model, and the bold (solid) line is the result obtained with the NLO QCD-evoluted CKMT model when one takes $Q_0^2 = 2.GeV^2$ ($Q_0^2 = 4.GeV^2$).

Figure 6. Gluon density distribution as a function of Q^2 calculated by performing the NLO QCD evolution of the CKMT model, and compared with the H1 Collaboration data in reference [20]. $g(x, Q^2)$ is plotted and not $xg(x, Q^2)$, in order to show more clearly the evolution with the scale. In the theoretical calculation, the bold (solid) line has been obtained by taking a value of Q_0^2 at the starting point of the QCD evolution, $Q_0^2 = 2.GeV^2$ ($Q_0^2 = 4.GeV^2$).

Figure 7. Gluon densities at $\mu^2 = 25.GeV^2$ as a function of x calculated by performing the NLO QCD evolution of the CKMT model, and compared to those determined from H1 DIS data (black dots) and from H1 photoproduction data (stars). Experimental data on D^* meson cross-section measurements are from references [16, 20]. In the theoretical calculation, the solid (dotted) line corresponds to a value of Q_0^2 at the starting point of the QCD evolution, $Q_0^2 = 2.GeV^2$ ($Q_0^2 = 4.GeV^2$).

Figure 8. Gluon density at $\mu^2 = 200.GeV^2$ as a function of x calculated by performing the NLO QCD evolution of the CKMT model, to be compared with that obtained from the analysis of the H1 di-jet data [16, 21]. In the theoretical calculation, the solid (dotted) line has been obtained by taking a value of Q_0^2 at the starting point of the QCD evolution, $Q_0^2 = 2.GeV^2$ ($Q_0^2 = 4.GeV^2$).

Figure 9. Prediction of the behavior of $xg(x, Q^2)$ as a function of x for several values of Q^2 measured both by H1 and ZEUS collaborations. The experimental points are not shown since the analysis of the more recent data is not completed. The solid (dotted) lines have been obtained by taking a value of Q_0^2 at the starting point of the QCD evolution, $Q_0^2 = 2.GeV^2$ ($Q_0^2 = 4.GeV^2$).